Learning Enhancement Team

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Worksheet: Adding and Subtracting Algebraic Fractions

This worksheet has questions that will test your ability to add and subtract algebraic fractions (fractions with *x*'s or *y*'s or other letters in them). The method is no different to the method used to add and subtract numerical fractions. You must first find a common denominator, then make the denominators the same in both fractions, only then can you add or subtract the numerators. Remember, **you cannot add or subtract fractions unless the denominators are the same**.

Adding and Subtracting Algebraic Fractions study guide	Model answers to this sheet

1. Add or subtract the following:

(a) $\dfrac{1}{x} + \dfrac{1}{x}$

(b) $\dfrac{1}{x} - \dfrac{1}{x}$

(c) $\dfrac{1}{x} + \dfrac{y}{x}$

(d) $\dfrac{1}{3x} + \dfrac{4}{3x}$

(e) $\dfrac{5}{2a} - \dfrac{1}{2a}$

(f) $\dfrac{1}{1+x} + \dfrac{7}{1+x}$

(g) $\dfrac{3x}{p+q} - \dfrac{2x}{p+q}$

(h) $\dfrac{s+t}{2u-v} + \dfrac{s-t}{2u-v}$

(i) $-\dfrac{1}{c^2} + \dfrac{1}{c^2}$

2. For these questions you need to find a common denominator (remember that you cannot add/subtract fractions which have different denominators). Here the common denominators are found by multiplying together the denominators in the question.

   Add or subtract the following fractions. None of them can be cancelled down.

(a) $\dfrac{1}{x} + \dfrac{1}{y}$

(b) $\dfrac{1}{x} - \dfrac{1}{y}$

(c) $\dfrac{a}{b} + \dfrac{c}{d}$

(d) $\dfrac{4}{x} + \dfrac{3}{4y}$

(e) $\dfrac{7}{x+1} - \dfrac{5}{y+1}$

(f) $\dfrac{p}{p-q} - \dfrac{a}{a+b}$

(g) $\dfrac{v}{2u+v} - \dfrac{1}{s}$

(h) $\dfrac{uvw}{abc} + \dfrac{xyz}{def}$

(i) $\dfrac{1}{x^3} - \dfrac{2}{y^2}$

3. Multiplying together the denominators always gives a usable common denominator. However if the two denominators have common factors you can find a simpler common denominator by finding the common factors of them and then multiplying this by the remaining factors. This can be thought of as the **simplest common multiple** of the two denominators. For example the simplest common multiple of $4t$ and $2t$ is $4t$.

Find the simplest common multiples for the following expressions:

(a) $2x$ and 2

(b) $2x$ and x

(c) x^2 and x

(d) 9 and 3

(e) abc and ab and a

(f) a^2bc^5 and $a^4b^2c^3$

4. If you do not use the simplest common denominator then you will still get the right answer, but your answers will have to be cancelled down. Therefore it is more effective to use the simplest common multiple when adding or subtracting fractions.

Using the simplest common multiples you found in question 3 to calculate:

(a) $\dfrac{1}{2x}+\dfrac{1}{2}$

(b) $\dfrac{1}{2x}+\dfrac{1}{x}$

(c) $\dfrac{1}{x^2}+\dfrac{1}{x}$

(d) $\dfrac{7y}{9}-\dfrac{x}{3}$

(e) $\dfrac{2d}{abc}-\dfrac{3f}{ab}+\dfrac{1}{a}$

(f) $\dfrac{5}{a^2bc^5}-\dfrac{1}{a^4b^2c^3}$

5. Remember you may only cancel down a fraction if the same letter or number is a factor of both the numerator *and* the denominator. For example $\dfrac{2+x}{2x}$ cannot be cancelled down as neither 2 nor x are factors of the numerator.

Bearing this in mind, add or subtract:

(a) $\dfrac{1}{2}+\dfrac{1}{x}$

(b) $\dfrac{1}{x+1}-\dfrac{1}{2}$

(c) $\dfrac{1+x}{2x}+\dfrac{x-1}{x}$

(d) $\dfrac{x-1}{x+2}-\dfrac{x-3}{x+1}$

(e) $\dfrac{ab}{c^2d^3}-\dfrac{a}{bc^2d}$

(f) $\dfrac{ab}{c^2d^3}-\dfrac{a}{bc^2d}+\dfrac{b}{c^2d^2}$

(g) $\dfrac{x}{y}+\dfrac{y}{x}$

6. When adding a non-fraction to a fraction, it may be helpful to think of the non-fraction as a fraction with denominator 1.

    Add the following:

    (a) $1 + \dfrac{1}{x}$
        (b) $x + \dfrac{1}{2}$
        (c) $5x + \dfrac{1}{5x}$
        (d) $x + 1 + \dfrac{1}{x}$

7. [Harder] Which of the following expressions are equivalent?

    (a) $\dfrac{1}{x} + \dfrac{1}{y}$
        (b) $1 + \dfrac{y}{x}$
        (c) $\dfrac{x+y}{x}$

    (d) $\dfrac{y+x}{y}$
        (e) $\dfrac{x+y}{xy}$
        (f) $2 + \dfrac{x-y}{y}$

8. [Harder] Add the following:

    (a) $\dfrac{1}{x} + \dfrac{1}{x^2}$
        (b) $\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3}$
        (c) $\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} + \ldots + \dfrac{1}{x^n}$