Non-Euclidean Crochet

**Skill and Experience** ●◑○○○

These are easy pieces to make, but you have to keep careful count! You just need to be able to do treble and double crochet for the shawl. Only double is needed for the three surfaces.

How it relates to maths

Imagine a two dimensional surface containing a line L and a point A which is not on L. How many lines can be drawn through A which are parallel to L?

If you look at the picture it may seem obvious that you can only draw one. This is true if the surface is a Euclidean surface, but there are other types of surfaces and other ways to think of points and lines.
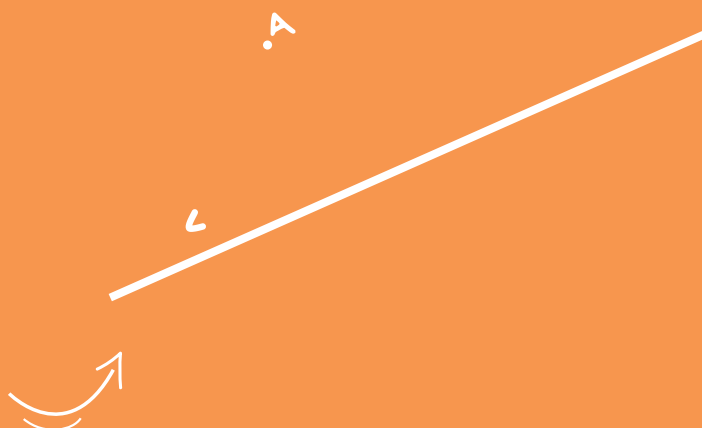
In a hyperbolic surface there are an infinite number of lines through A which are parallel to L. In an elliptic surface there are none.

The Poincaré Disc model of a hyperbolic surface is a two-dimensional disc (shown as the inside of a dotted circle in the diagram) where lines are represented as arcs of circles which meet the edge of the disc at right angles. The edge of the disc represents infinity and distances between points are dependent on where they are in relation to the centre of the disc.
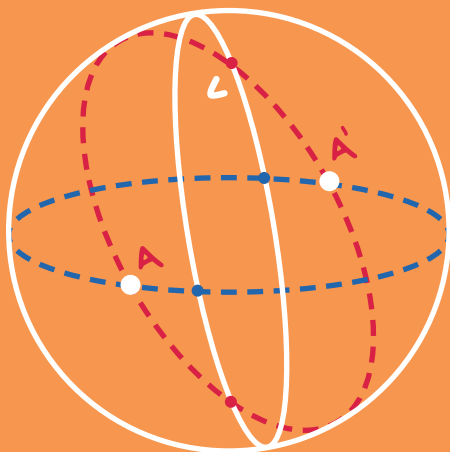
Four straight lines on a Poincaré disc
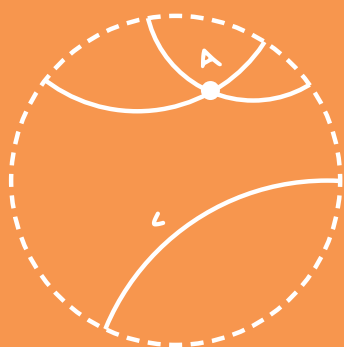
Two straight lines through point A which are parallel to line L

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An elliptic surface can be modelled by the two-dimensional surface of a sphere. Every circle on a sphere that has its centre at the centre of the sphere is a straight line (they are "great circles", like the equator). Now no two straight lines can be parallel as every great circle intersects every other great circle. The problem now is that two straight lines do not intersect at a single point. Every two lines intersect at two points on exactly opposite side of the sphere. So we define a 'point' in this surface as the pair of opposite points on the sphere.

It is possible to make a surface by starting with *a* stitches and letting the *nth* row have ak^{n-1} stitches (rounded to the nearest integer!). If *k* is 1, the surface is Euclidean and anything drawn on it will be like something drawn on ordinary paper. If *k > 1* the surface is hyperbolic and looks quite organic and like coral. If *k < 1*, the surface is elliptic. If you start with a straight line of crochet and increase the number of stitches exponentially, so that each row has $\frac{12}{11}$ times the number of stitches in the previous row, it turns into a very organic looking shawl like this:

Two straight lines on an elliptic surface intersecting at a single (double) point.
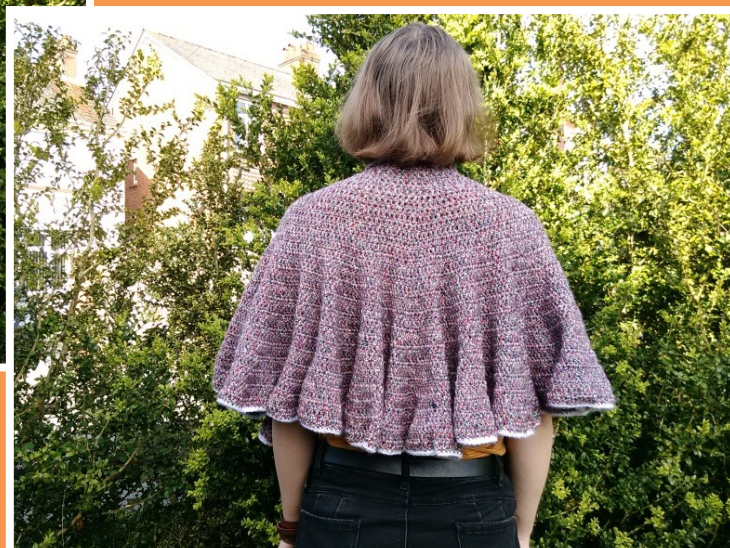
Point AA' is not on line L, but every line through AA' passes through L.

Hyperbolic surfaces can be created in crochet. I first came across the idea via the work of Daina Taimina (**www.hyperbolic-crochet.blogspot.com**). It is all tied up with exponential growth.

However, if you start with a straight line of crochet and decrease the number of stiches exponentially, so that each row has $\frac{11}{12}$ times the number of stitches in the previous row, it makes something which looks like a deformed pizza slice with the mozzarella stretching away. It doesn't make a good shawl!

These are interesting and good fun to make, but they don't seem to have any relation to the mathematical models I started with. Now think about how infinity is shown in a finite model. In the Poincaré model infinity is all around the edge of the disc, but on my hyperbolic shawl infinity is only in one direction; the bit where I stopped and did my white edging. A better representation would be to crochet a circle.

To make a Euclidean surface you need to increase the number of stitches in each circular row depending on how far from the centre you are. A simple model is to take each stitch as a unit, so you have 2π in the first row, 4π in the second row, 6π in the third and $2n\pi$ in the nth row.

This can only be approximate as you have to have integer values (you can't make 6.283185 stitches!)

To make a non-Eucildean surface the *nth* row has k^{n-1} times more stitches than the *nth* row of the Euclidean version. Adapting this idea to the Euclidean circle where the nth row has $2n\pi$ stitches gives a surface with $2n\pi\, k^{n-1}$ stitches for the nth row. It is a hyperbolic surface if $k > 1$, or an elliptic surface if $k < 1$.

More about how these crocheted surfaces relate to the Poincaré disc and sphere model is given in the file "Relationship to the models".

Hyperbolic Shawl

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What you'll need

500g double knit yarn

50g double knit yarn in a contrasting colour

4mm crochet hook

A tapestry, darning or wool needle

Scissors

Step-by-Step Instructions

1 Make a slip knot then a chain of 36 stitches. Turn and start working treble stitch (called double in the US) in the next but one from the loop the hook is in, so that you make 35 stitches.

2 Turn the work and work treble stitch but every 11th stitch work two trebles into the same stitch (so the second row has 38 stitches).

3 Keep doing this, working two stitches into every 11th stitch. In order to hide the increase, look at the previous row and see where the double stitches are. For example, in the first row where you do increases you finish the row with two stitches in the last stitch. So if you work, say, four stitches and then do the first increase and every 11th stitch after that, the increases will not follow a pattern.

4 Keep working two stitches into every 11th stitch until you do not have enough yarn to complete the next row.

5 Work a row of double crochet (called single in the US) in your contrasting colour (I used white) and sew in ends.

Quick Tips

For a longer shawl use more yarn. If you do make it longer, you might want to make it grow a little less quickly or you could end up needing a whole ball of yarn for a single row. Try increasing every 20th stitch perhaps, or every 30th stitch.

If you have difficulty keeping count, you could put a stitch marker or safety pin in every time you increase so you can see where they are in the previous row. This way, you can easily count on when you put the work down and pick it up again a few days later.

Adapt the idea any way you like. There are no crochet police! Do your own thing.

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Euclidean Surface

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What you'll need

- 50g Aran weight yarn
- A copy of the sheet called "Circular start"
- 5.5mm crochet hook
- A pen or pencil
- Stitch marker or safety pin
- A tapestry, darning or wool needle
- Scissors

Step-by-Step Instructions

1 Make a paper copy of "Circular Start".

2 Make a magic ring and work six double crochet stitches into it (UK double = US single). Tighten the magic ring.

3 Put a marker in to show where the first row ends.

4 Looking at the "Euclidean k=1" column, cross off the first number "6".
Look at the next number for row two. It is "13", so you need to get 13 stitches into the next row. That means making three stitches into one stitch of the first row and two stitches in the other five stitches. Move the marker to the end of this row.

5 Cross off the number 13 and look for the number of stitches needed in the next row. Row three needs 19 stitches. That is 6 more stitches to fit into 13, so make the next row with alternating one and two double stitches into each stitch of the previous row until you come around to the marker. Move the marker to the end of this row.

6 Cross off the number 19 and look for the number of stitches needed in the next row. Row four needs 25 stitches. That is six more stitches to fit into 19, so crochet around but put two stitches into every third stitch until you come around to the marker. Move the marker to the end of this row.

7 Keep working down the list until you feel your surface is big enough, or you run out of yarn. If you want more than 45 rows, use the formula $y=2x\pi$ to calculate the number of stitches, rounding your answers to the nearest integer.

Quick Tips

There are lots of tutorials on how to make a magic ring to start circular crochet on YouTube. You can also start by working six stitches into a slip knot.

The marker is needed because you are working a spiral so it is next to impossible to work out where the next row starts if you don't mark it.

If you want to crochet a surface with more than 45 rows, the formula is given at the top and numbers are simply rounded to the nearest integer.

Notice that the number of stitches to increase by is always close to six. It is not exactly six more stitches each row, sometimes it is seven. This is because pi is not exactly three. It is a bit like having an extra day every leap year.

As the circle gets bigger, try to space the added stitches around the work to avoid them being too close to the added stitches in the previous row. You have to jiggle things a bit to fit them in when there are seven to add in.

NON-EUCLIDEAN CROCHET

Hyperbolic Surface

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What you'll need

50g Aran weight yarn

A copy of the sheet called "Circular start"

5.5mm crochet hook

A pen or pencil

Stitch marker or safety pin

A tapestry, darning or wool needle

Scissors

Step-by-Step Instructions

1 Make a paper copy of "Circular Start".

2 Make a magic ring and work six double crochet stitches into it (UK double = US single). Tighten the magic ring.

3 Put a marker in to show where the first row ends.

4 Looking at the "Hyperbolic k=21/20" column, cross off the first number "6". Look at the next number for row two. It is "13", so you need to get 13 stitches into the next row. That means making three stitches into one stitch of the first row and two stitches in the other five stitches. Move the marker to the end of this row.

5 Cross off the number 13 and look for the number of stitches needed in the next row. Row three needs 21 stitches. That is eight more stitches to fit into 13, so make the next row with alternating 1, 2, 2 double stitches into each stitch of the previous row until you come around to the marker (1 2 2 1 2 2 1 2 2 1 2 2 1). Move the marker to the end of this row.

6 Cross off the number 21 and look for the number of stitches needed in the next row. Row four needs 29 stitches. That is eight more stitches to fit into 21, so crochet around putting two stitches into eight of the previous row stitches as evenly spaced as you can. When you come around to the marker, move it to the end of this row.

7 Keep working down the list until you feel your surface is big enough, or you run out of yarn. If you want more than 45 rows, use the formula $y = 2x\pi \left(\frac{21}{21}\right)^{x-1}$ to calculate the number of stitches, rounding your answers to the nearest integer.

Quick Tips

You can either make a list of where you are going to add the extra stitches in each row before you start, or just do it by eye.

This doesn't have to be made in Aran weight yarn. That was just what I decided to use because I had some cream yarn to hand and I knew I might want to embroider points and lines onto the surfaces when they were done. These surfaces could also be made in double knitting with a 4mm hook.

You DO need to keep track of how many extra stitches you have added so far in a row. Something written down is safer than keeping it in your head, especially if you are likely to be interrupted in the middle of a row!

Elliptic Surface

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What you'll need

50g Aran weight yarn

A copy of the sheet called "Circular start"

5.5mm crochet hook

A pen or pencil

Stitch marker or safety pin

A tapestry, darning or wool needle

Scissors

Step-by-Step Instructions

1 Make a paper copy of "Circular Start".

2 Make a magic ring and work six double crochet stitches into it (UK double = US single). Tighten the magic ring.

3 Put a marker in to show where the first row ends.

4 Looking at the "Elliptic k = 19/20" column, cross off the first number "6". Look at the next number for row two. It is "12", so you need to get 12 stitches into the next row. That means making two stitches in each of the six stitches of the first row. Move the marker to the end of this row.

5 Cross off the number 12 and look for the number of stitches needed in the next row. Row three needs 17 stitches. That is five more stitches to fit into 12, so space them out as best you can until you come around to the marker (perhaps 1 2 1 1 2 1 2 1 1 2 1 2). Move the marker to the end of this row.

6 Cross off the number 17 and look for the number of stitches needed in the next row. Row four needs 22 stitches. That is four more stitches to fit into 17, so crochet around putting two stitches into four of the previous row stitches, as evenly spaced as you can. When you come around to the marker, move it to the end of this row.

7 Keep working down the list until you reach row 17. This row is the same size as the previous row, so you do not do any increase stitches, but make 47 stitches and move the marker to show you have finished a row. Do this again and again until you have completed the 23rd row.

8 Row 24 has one stitch less than row 23, so you need to decrease by one stitch somewhere in the row.

9 Keep working down the list. You are likely to want more than 45 rows, so use the formula $y=2x\pi\left(\frac{19}{20}\right)^{x-1}$ to calculate the number of stitches, rounding your answers to the nearest integer.

Quick Tips

Try to space the increase and decrease stitches out around the rows.

If you do not know how to decrease in double crochet look up "single crochet decrease" on YouTube. Remember that what is called double crochet in the UK is called single crochet in the US, and most YouTube tutorials on crochet are made in the US.

This surface is more lumpy and inaccurate than the hyperbolic surface because the numbers get small, so rounding to integers gives bigger errors.

I found my crocheted elliptic surface got a bit twisty because of the rounding errors and I had to stop at 4 stitches per row because I was losing where the loops were and couldn't see well enough to decrease any more.
In theory the tail could go on towards infinity, getting thinner and thinner. A smoother picture of the shape can be seen on the Autograph file "Elliptic Surface".

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