Some questions will be straightforward tests of the mathematics you know. These questions will probably be like the ones you do at school.

1 Sami buys 25 mugs for £0·84 each. He gives away 4 of them, and sells the rest at £1·40 each. What percentage profit does he make?

2 A *Finest* cream dessert is sold in tubs of 450 ml which contain 125 ml of cream.
A *Superb* cream dessert is sold in tubs of 375ml which contain 105 ml of cream.
Which cream dessert contains the greater proportion of cream?

3 Solve the equations.

a $\frac{1}{6}x - \frac{1}{4}(x-5) = 1$

b $\dfrac{x}{x-1} = \dfrac{x+1}{x-2}$

Some questions might be puzzles, which just expect you to be prepared to play intelligently with numbers:

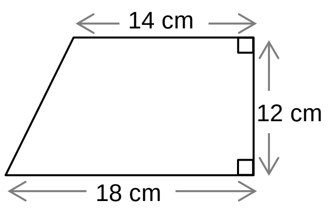
4 The five-digit numbers 91723 and 85604 use all ten digits between them. The difference between these numbers is 91723 – 85604 = 6119.
Find two five-digit numbers which use all ten digits between them and which have the *smallest possible* difference.

5 Find three different whole numbers A, B and C, so that

- B is the average of A and C
- A^2 is the average of B^2 and C^2.

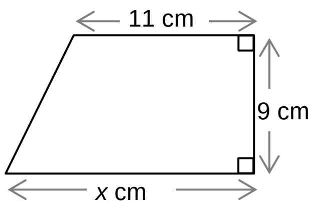
Note that not all of the numbers can be positive.

Some questions might expect you to work backwards through a problem.
For example, you could probably work out the area of this trapezium easily enough



but question **6** *asks you to work backwards through a similar problem.*

6 The area of this trapezium is 117 cm^2.What is the length x cm of its base?

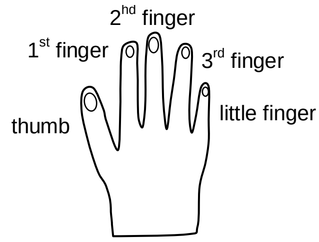


Here is a harder example.

7 From January 1st 2004 until January 1st 2013, average rental prices in London rose by 50%. From January 1st 2004 until January 1st 2007, average rental prices in London rose by 20%. By what percentage did average rental prices in London rise from January 1st 2007 until January 1st 2013?

Some questions might expect you to explain your reasoning, as well as just give an answer.

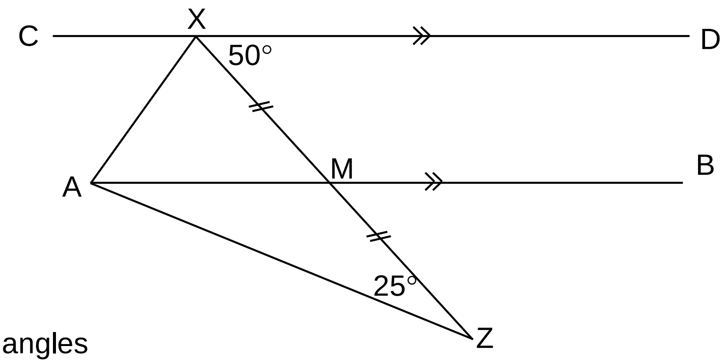
8 A boy counts on his fingers, backward and forwards across his right hand as follows: thumb, 1st finger, 2nd finger, 3rd finger, little finger, 3rd finger, 2nd finger, 1st finger, thumb, 1st finger, … and so on.



If he starts counting at one, on his thumb, which finger will he be on when he reaches two thousand and thirteen?
Explain clearly how you decided.

This will certainly be true of questions about Geometry, using the triangle and parallel line theorems and the properties of polygons.

9 In the diagram line CD is parallel to line AB, and M is the midpoint of line XZ.
Angle DXM = 50° and angle MZA = 25°.



Find angles
i MAZ
ii CXA.
showing and explaining each step in your working.

You might also have to decide about a statement whether you can be sure it is true, it might or might not be true or it definitely isn't true, with an explanation.

10 You are given that N is a whole number which is a multiple of 6 and a multiple of 10.
For each of the following statements say whether you can be sure it is true, it might or might not be true or it definitely isn't true. Explain your answer in each case.
a N is a multiple of 60.
b N has a factor of 15.
c N is a factor of 100.
d N is a multiple of 7.

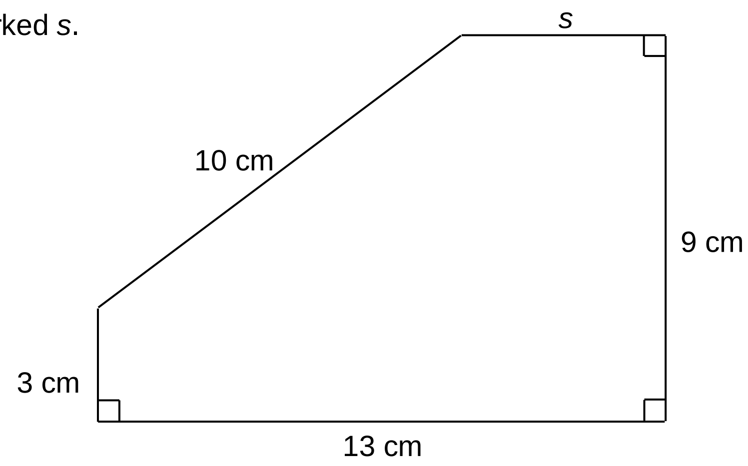
In some questions, you might have to make more than one step to get from the information you are given to the answer you are asked for.

11 In a school, there are 47 pupils in year ten.
Nine girls are in the netball squad and twelve girls are in the athletics squad. Three girls are in both squads and seven girls are in neither squad.
Thirteen boys are in the football squad and eight boys are in the basketball squad. Five boys are in neither squad. How many boys in year ten are in the football squad and the basketball squad?

This may also involve adding additional lines to a diagram you are given.

12 The diagram shows an irregular pentagon. The lengths of four of the sides are shown in the diagram. Three of the angles in the pentagon are right angles, as shown.

Find the length of the side marked *s*.

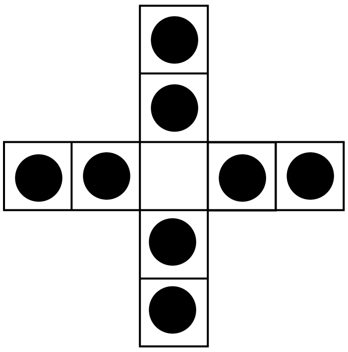


Sometimes, there will be a lot of information in the question, and you have to organise it for yourself, to see what information is relevant in each step.

13 A man is going to ride his horse from his house to a town 39 miles away. After he leaves his house, he rides at 24 miles per hour for the first forty-five minutes, then speeds up when he reaches a good track and rides at 36 miles per hour for the next 15 miles, until he reaches a river. He then lets the horse drink for ten minutes. After this, he continues to town at a gentle pace through the woods. Altogether, he finds he has taken two hours to travel the 39 miles from his house to the town. What is his average speed when he is riding at a gentle pace through the woods?

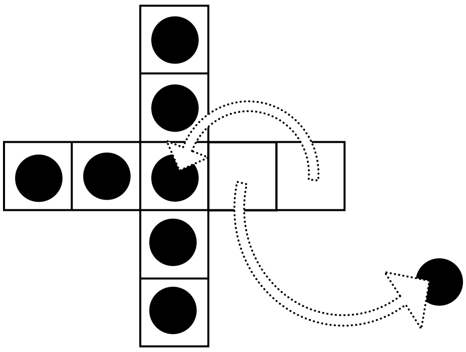
Some questions might give you a set of rules and ask you to work out what will happen: in a game, for example.

14 The diagram shows nine squares. There are counters on eight of the squares; the ninth square is empty.



A move on this diagram consists of jumping one counter over another to an empty square. The jumping counter moves in a straight line (up, down, left or right) and lands exactly two squares away from its original position. The counter that is jumped over is removed.

For instance, after the first move the position could be



a Show that all four possible first moves lead to a rotation or reflection of the same position.

The game ends when it is impossible to make a move.

b Show that, however you continue this game, there are at least two counters left when the game ends.

Other, probably longer, questions might describe a new situation, and ask you to learn about it by investigating for yourself. It is likely that, in these questions, the early parts will be quite easy, while you work out what is going on, and the later parts will require you to think quite hard about what you have found out.

15 To **double** a number means to multiply it by two.
In this question, to **twiddle** a number will mean to add four to it, and to **flip** a number will mean to subtract it from 8 (so **flip**ping 3 gives 5, **flip**ping –1 gives 9 and so on).

a If you **twiddle** a number and then **twiddle** the answer, the overall effect of the two operations is to add eight to the number. What would be the overall effect of the two operations if you:

i **flip** a number and then **flip** the answer?

ii **twiddle** a number and then **flip** the answer?
iii **flip** a number and then **twiddle** the answer?

b Show that if you **twiddle** a number, **flip** the answer and then **twiddle** the answer to that, this has the same overall effect as just **flip**ping the number once.

c Find a sequence of three operations, each a **twiddle** or a **flip**, that you can carry out on any number and which has the overall effect of just changing the sign of the number.

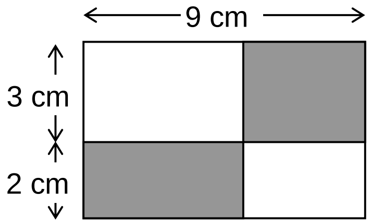
d **i** Find a sequence of six operations, each a **twiddle** or a **flip**, that has the overall effect of leaving every number you apply it to unchanged.
ii Is it possible to find an odd number of operations, each a **twiddle** or a **flip**, that has the overall effect of leaving every number you apply it to unchanged? Justify your answer.

Some questions will ask you to use the algebra that you have learnt. Question **16** *is quite easy and you will probably have done questions like this at school, question* **17** *is harder. In question* **18***, the difficulty is spotting what to call "*x*".*

16 Five bananas and two kiwi fruit cost £2·30, and four bananas and three kiwi fruit cost £2·47. How much would it cost to buy two bananas and one kiwi fruit?

17 At the start of the year, there were three times as many boys as girls in a class.
At Christmas, two new girls joined the class and one of the boys was expelled.
Then there were only twice as many boys as girls in the class.
How many boys and girls were there in the class at the start of the year?

18 The two shaded rectangles have equal area. What is the total shaded area?



Finally, there might be questions that teach you an idea or a technique, and ask you to use it for yourself in a slightly different problem.

19 PROBLEM Find x, y and z if
$x + y = 5$
$z + y = 11$
$x + z = 13.$

METHOD The first equation can be rearranged to
$x = 5 - y.$
and the second equation can be rearranged to
$z = 11 - y.$
These two rearrangements imply that
$z + x = (5 - y) + (11 - y) = 16 - 2y$
but the third equation says that
$z + x = 13,$

so

$$16 - 2y = 13.$$

This is a straightforward equation which can be solved to give $y = 1\frac{1}{2}$.

The first step of this METHOD can now be used to conclude that

$$x = 5 - y = 5 - 1\tfrac{1}{2} = 3\tfrac{1}{2}$$

and

$$z = 11 - y = 11 - 1\tfrac{1}{2} = 9\tfrac{1}{2}.$$

a Use the METHOD above to solve the PROBLEM

Find x, y and z if

$$x + y = 12$$
$$z + y = 17$$
$$x + z = 21$$

b Adapt the METHOD to solve the PROBLEM

Find x, y and z if

$$x + 3y = 14$$
$$z + 5y = 21$$
$$2x - z = 5$$

c Try to adapt the METHOD to solve the PROBLEMs below. What goes wrong?

i Find x, y and z if

$$x - y = 15$$
$$z - y = 6$$
$$x - z = 11$$

ii Find x, y and z if

$$x - y = 15$$
$$z - y = 6$$
$$x - z = 9$$

d One of the PROBLEMs in **c** has many solutions, even though the METHOD does not work; but the other PROBLEM has no solutions.
Identify which problem is which, and explain what the difference is between the two PROBLEMs which means that one has solutions and the other does not.