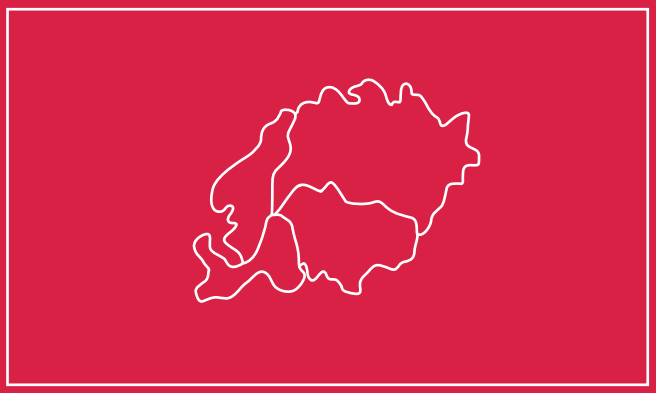
EXETER MATHS SCHOOL

# Seven Colour Torus Puzzle

**Skill and Experience**

## How it relates to maths

In 1852 a student, called Francis Guthrie noticed that you only need four colours to colour a map of England so no two counties of the same colour meet in an edge. In fact, you only need four colours to colour in ANY map. Sometimes you don't need all four colours, but you never need more.

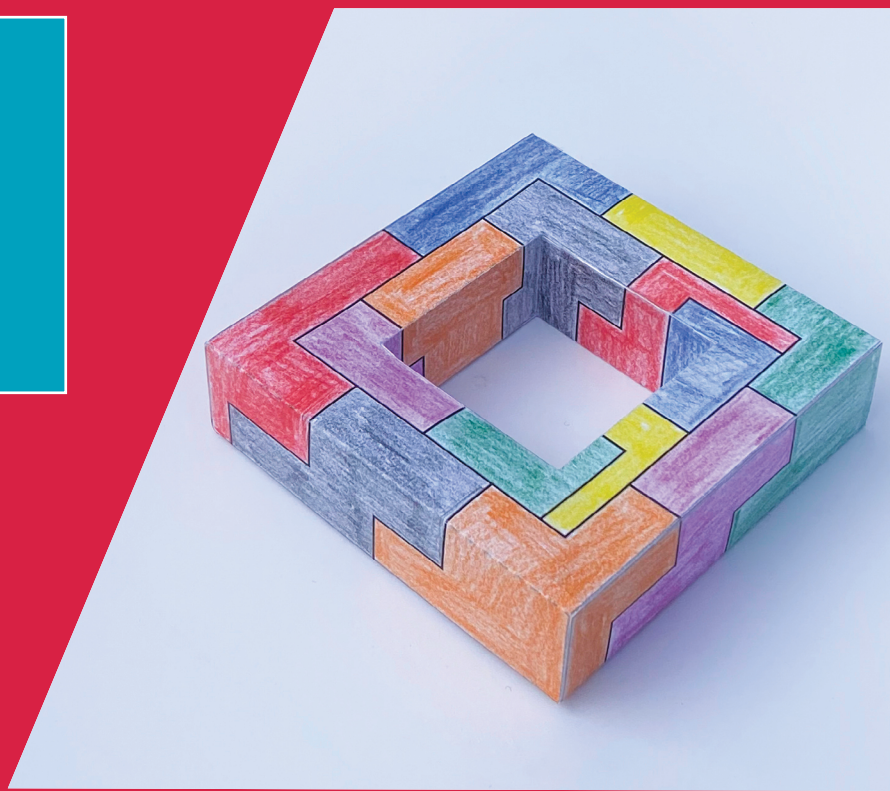


This map can be coloured in with just three colours.

If I colour in the sea around the island in blue, I can colour the four sections in the island in two colours, say red and yellow, to look like this:

Can you see that this is actually the same map?

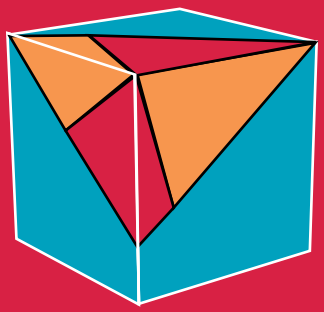
It just has all the wiggles smoothed out, and it is in a circle rather than in a rectangle. You could colour it in just the same way as the previous one. The area of mathematics where these two maps are considered to be the same is called topology.



This next map needs four colours, but no more. Try copying it out and colouring it. There is a good online set of 40 maps to colour here: **Play Four Color Theorem - Coloring Puzzle Game, a free online game on Kongregate**.

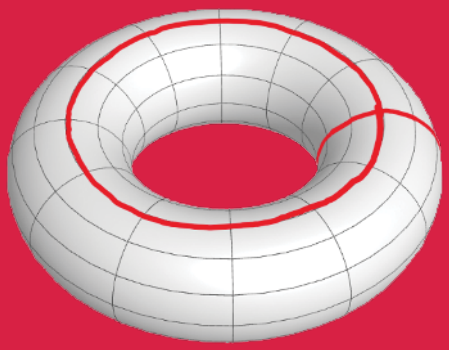
I wrote “you only need four colours to colour in ANY map”. This is not quite true. It depends what you draw the map ON. Imagine the island in the sea being on a sphere rather than flat in a rectangle. Any map drawn on a sphere will never need more than four different colours.

Just as a square and a circle are the same in topology, because each is a single loop with an inside and an outside, so a sphere is the same as a cube. Imagine a cube made of rubber. If you pumped air into it, it would blow up into a sphere. Here is the same map again:

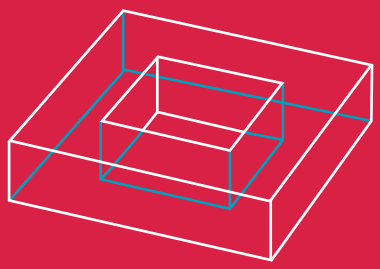


BUT if you draw a map on a torus, four colours is not enough!

A torus is like a ring doughnut. Because there is a hole in the torus it is possible to draw two circles which intersect only once. The two circles drawn on the torus here do NOT make a map. There is only one “country”, so the lines do not make a border between countries. If you imagine you are an ant on the torus, you can reach every part of the torus without crossing a border line.



Just as a sphere and a cube are the basic shape, so the torus above is the same shape as this rectangular torus below.



It iis possible to draw a map on a torus where no country has a border which meets itself and more than four colours are needed. The puzzle in this section is a map on a torus with seven countries and every country touches every other country, so SEVEN colours are needed to colour it!

SEVEN COLOUR TORUS PUZZLE

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## What you'll need


A copy of "7 colour torus net" printed out on white card

Seven colouring pencils or pens of different colours

Optional: a craft knife to cut the inner lines really neatly.

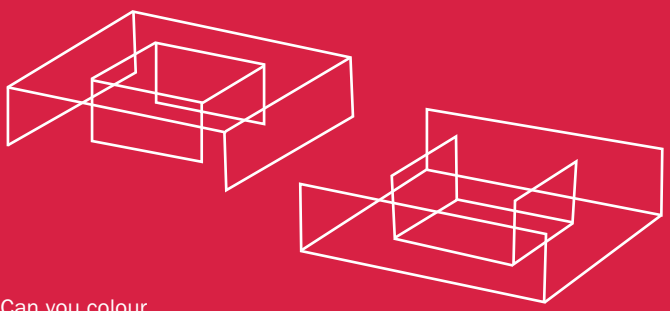
Scissors

## Step by Step Instructions

**1** Print out the "7 colour torus net" on white card.

**2** The black double lines will be the lines you cut. The grey dotted lines are the lines you will fold. The black solid lines are boundaries between the countries.

**3** **Imagine** cutting out (but don't cut yet!) the two parts of the torus, cutting the black double lines and folding the grey dotted lines to get two pieces which look like this:



**4** There is only one way the two pieces can fit together so that the edges of the countries match up. Remember that a country will continue over a fold line but may not continue over a cut line.

**5** Now the challenge! Can you colour in the 7 countries in seven different colours, continuing from one piece of card to the other?

**6** When you have coloured it all in, cut it out and put it together to see if you are right. The two pieces can be stuck together with sticky tape. Each country should wind around the torus and touch every other colour.

## Quick Tips

If it just feels too hard to do, do the easier challenge of cutting out the two shapes first, fold them and fit them together so the edges of the countries agree, and then colour it in.

The H cut in the middle of each rectangle is very hard to do with scissors. When giving this to students I cut the H shape with a craft knife and metal ruler in advance.

I labelled the outer sides of the rectangles as I worked out what would touch what, so that I didn't have to keep working it out again as I got confused myself. An example of the colouring can be seen in the "solution" file.