

# Two Unzippable Klein Bottles

Skill and Experience 🔴 🔿 🔿 🔿

These are very easy objects to make which can be made very quickly on a sewing machine.

# How it relates to maths

There is an explanation of what a Klein bottle is in "The Klein Bottle and Projective Plane" and there is an explanation of what a Möbius strip is in "A Möbius strip with an intrinsic-twist".



A Klein bottle can be represented by a labelled square like this, so if we start with a rectangle of material and sew the two sides of a zip onto the "a" sides then twist the material and sew the "b" sides together, we must get a Möbius strip when the zip is undone and a Klein bottle when the zip is done up!

BUT, if you look online you will discover that if you chop a Klein bottle in half you get TWO Möbius strips with mirrored twists. There is a very good picture of this at the bottom of the page on this site: **The Klein Bottle – After Paul Chang (illinois.edu).** 

#### How can this be?

The labelled square definitely defines a Klein bottle and if you join the "b" sides you definitely get one Möbius strip. If I were to cut that Möbius strip up the middle I would NOT get two Möbius strips with opposite twists (try it with a paper Möbius strip if you haven't done so before!). But the story is consistent across the web – if you cut a Klein bottle you get two Möbius strips. How can they both be true?

I had to think hard to convince myself that both are possible. Basically the one Möbius strip version of the Klein bottle has the zip where the yellow line is on the left-hand diagram. Remember that the two "a" sides represent the same line once the surface is folded up, so there is only one zip. The "b" sides twist to connect, so dots of the same colour represent the same point on the Klein bottle. If we didn't leave a space for the bottle to pass through itself, the zip would finish at the point where it started (at the black point).



The red, blue and green dots represent three points on the "b" side. The right-hand diagram shows where the zip goes to cut the Klein bottle into two Möbius strips. It twists around the surface. Again, if we didn't leave a space for the bottle to pass through itself, the zip would finish at the point where it started (at the black point). Remember the blue dots both represent the same one point on the surface. The black, red and blue dots on the top of the square are the blue, red and black dots on the bottom, so you can see the strip in the middle is a Möbius strip when the surface is folded up. The bottom left triangle and the top right triangle of the square become a second Möbius strip when you connect the "a" sides.

If that still seems confusing, the best thing to do is to make the fabric Klein bottles for yourself, zip them up and unzip them and turn them around in your hands!





#### TWO UNZIPPABLE KELIN BOTTLES

### What you'll need





One square metre of fleece

82cm An open



An open ended zip 183cm long



Unzip the 82cm

two pieces.

zip so that it is in

A sewing machine



Tape measure

Top



way down the other end.

Sewing

thread



Scissors

Sew one side of the zip to each of the long sides of the

100cm x 50cm rectangle of material so that the top

of the teeth are a centimetre from the end and the zip

puller and the bottom of the zip are in line not all the



A needle to sew in ends

### **Step-by-Step Instructions**

ended zip

82cm long

**1** Cut the fleece into three pieces. One which is 100cm by 50cm and two which are 100cm by 25cm.



Twist the material and match the ends up, A with A and B with B as shown in the diagram. Lay the ends over each other and sew them together using a zigzag stitch.

Connect the zip and zip it up. Pull it around to convince yourself you have a Klein bottle.

6

Now make each 100cm x 25cm rectangle into a Möbius strip, sewing the ends together overlapping by 1cm with zigzag

stitch, but being careful to turn one clockwise and one anticlockwise so that you have a mirror image pair.

8



Open the 183cm zip part-way. Lay the two looped pieces of material down so they are symmetrical and pin the top of the zip just below the seam on each side so that it looks symmetrical.



7

**10** Sew in any ends to neaten up.



Connect the zip and zip it up. Pull it around to convince yourself you have a second Klein bottle.





## Quick Tips

I had some left over pieces of fleece but not enough to make both Klein bottles in the same colour. I had enough grey to make the first bottle but only enough of other colours to make half a Klein bottle, so my second one is white and yellow. Fleece is a good material to use because it looks approximately the same on both sides, it stretches a bit and does not fray. Any other material could be used, but you might need to hem it first. It can all be hand stitched rather than machine sewn, but it will take a lot longer. The zip must not go all the way to the end of the rectangle or Mobius strip so there is room for the intesection, which is necessary so the tube can pass through itself.

For both Klein bottle, sew the zips from the top to the bottom on both sides so that they align as exactly as possible.

